Evaluation of Mass Transfer Coefficient of Gas Absorption into Falling Liquid Film

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Abstract
The analytic, open-form solution for the steady-state, 2-dimensional partial differential equation describing the concentration profile for the gas solute in a falling liquid film flowing under a steady-state laminar condition was shown. The overall mass transfer coefficient, $\Gamma$, for a binary diffusion system, was evaluated in light of the analytic solution and was tested for oxygen absorption into falling water film. $\Gamma$ was expressed as a function of Reynolds number, Peclet number for mass transfer, and geometrical properties of the film. A case study of $O_2$/water system was examined to validate the mass transfer based on the derived model for $\Gamma$ versus that given by the bulk terms. It was found that the estimated value of $\Gamma$ predicted a mass transfer rate that has about 10 % percent relative error (PRE) compared with that given by the convective mass transfer.

SYMBOLS USED
- $C_s$: [mol L$^{-1}$] concentration of solute (dissolved gas) A in falling liquid film; $C_{si}$: [mol L$^{-1}$] interfacial concentration of A; $C_{si,i}$: [mol L$^{-1}$] inlet concentration of A; $C_{AI}$: [mol L$^{-1}$] the average outlet concentration of A; $D_{AI}$: [m$^2$ s$^{-1}$] diffusivity of solute A in medium B; $D_{ai}$: [m$^2$ s$^{-1}$] the molar flow of solute A; $n_{iy}$: [mol s$^{-1}$] the molar flow rate of A absorbing into the falling liquid film; $N_{Re}$: [dimensionless] Reynolds number; $N_{Pe}$: [dimensionless] Peclet number for mass transfer defined as: $N_{Pe} = N_{Re} \times N_{Sch} = \frac{\mu}{D_{ai}}$; $n_{iy}$: [dimensionless] Schmidt number defined as: $N_{Sch} = \frac{\mu}{\rho D_{ai}} = \frac{\mu}{\rho} \frac{v}{D_{ai}} = \frac{\nu}{D_{ai}}$; $u_{iy}$: [m s$^{-1}$] the y-component velocity of the falling liquid; $W$: [m] the width of the falling liquid film.

GREEK SYMBOLS
- $\Gamma$: [kg m$^{-1}$ s$^{-1}$] the mass velocity (i.e., mass flow rate of falling liquid per unit width of solid boundary); $\delta$: [m] the liquid film thickness; $\eta$: [dimensionless] defined as: $\eta = \frac{\frac{1}{3} \sum N_{Pe} \frac{\delta}{L}}{N_{Pe}}$; $\theta$: [dimensionless] dimensionless concentration of A, defined as: $\theta = \frac{C_{si} - C_{i}}{C_{AI} - C_{i}}$; $\nu$: [kg m$^{-1}$ s$^{-1}$] the dynamic viscosity of the falling liquid; $\nu$: [m$^2$ s$^{-1}$] the kinematic viscosity or moment diffusivity of the falling liquid; $\rho$: [kg m$^{-1}$ s$^{-1}$] the density of the falling liquid; $\tau$: [N m$^{-2}$] The shear stress which represents the force in the j$^{th}$ direction divided by the i$^{th}$ unit surface area.

INTRODUCTION
Thin falling liquid film, by gravity, has numerous applications; for example, wetted-wall absorbers, falling film chemical reactors, condensers, and vertical tube evaporators. Falling film absorbers may be used for dissolving gases in liquids, separating gas mixtures, and removing unwanted species from a gas flow. A reliable design of such pieces of equipment mainly depends on how accurately the designer will be able to predict the transport rates of heat and mass to the flowing film.

Chermiti et al. [1] analytically derived and graphically presented expressions of the liquid velocity, the gas concentration, the entropy generation rate as well as the main sources of irreversibility in the case of gas absorption (carbon dioxide) into a laminar falling viscous incompressible liquid film (water) without chemical reaction. Generally, their results showed that longitudinal mass transfer flux and transverse mass transfer flux decreased from the interface and dropped towards zero value near the liquid film diffusion thickness $\delta$. From the interface towards diffusion thickness, it was found that entropy generation increased towards a maximum value due to diffusive mass transfer, then decreased towards a minimum value by convective mode.

Xu et al. [2] examined mass transfer across the falling film using both simulations and experiments. Based on the validity of the
simulated results for wave parameters, numerical experiments for mass transfer were carried out with the aim of comparing to the empirical relation based on a single hydrodynamic parameter, $\beta$, (the gradient of the vertical fluctuating velocity at the interface, $s^+$). They concluded that $\beta$ could be the all-important near surface hydrodynamic parameter governing the scalar transport across the interface. They suggested that the order of magnitude of the term: 

$$k_{L,mean}Sc_{0.5} = C \sim O(1.0)$$

where, $k_{L,mean}$ is the average liquid side mass transfer velocity; $Sc$ Schmidt number, and $\nu$ the kinematic viscosity.

Raisul Islam et al. [3] examined simplified models for coupled heat and mass transfer in falling-film absorbers. They compared the accuracy of a linearized coupled model for the heat and mass transfer in falling-film absorbers with a non-linear model. Under certain conditions, they found that the linearized model may be reduced to the traditional logarithmic mean difference (LMD) formulation. The analytical expressions resulting from the linearized model, however, were used to extract the heat and mass transfer coefficients from the experimental data for a horizontal tubular absorber and a vertical tube absorber. The mass transfer coefficients obtained using the linearized model and the LMD formulation agreed within about 10% while the heat transfer coefficients showed large differences.

Demirel and Sandler [4] examined the effects of concentration and temperature on the coupled heat and mass transport using published experimental data on heat and mass transfer aspects of binary and ternary liquid hydrocarbon mixtures. Using the linear non-equilibrium thermodynamics (LNET) and the dissipation-phenomenological equation (DPE) approach, they managed to assess the effects of concentration, temperature, molecular weight, chain-size, solute, solvent, and degree of branching on the degree of coupling between heat and mass flows.

Demirel and Sandler [5] exploited the linear non-equilibrium thermodynamics (LNET) approach, combined with the phenomenological theory, to express the entropy generation and dissipation functions representing the true forces and flows for heat and mass transport in a multi-component fluid. The LNET theory utilizes the dissipation function or the entropy generation function, based on the Gibbs relation, which assumes that local thermodynamic equilibrium holds for pseudo-equilibrium processes. They have expressed the phenomenological equations (PEs) with the resistance coefficients that are capable of reflecting the extent of the interactions between heat and mass flows. The use of the resistance type of the phenomenological coefficients (PCs) in PEs, in which the conjugate forces and flows are identified by the dissipation function were denoted by the authors as the dissipation-phenomenological equation (DPE) approach.

Leuthner et al. [6] studied heat and mass transfer of an evaporating wavy falling film of a water–ethylene glycol mixture. Transport rates could be predicted by a numerical procedure, including flow, temperature and concentration field. Their model included wave parameters, like base length of the average wave and mean height of wave above wall. The accuracy of the prediction compared with measurements was reasonably good at Reynolds numbers above 250. For their binary mixture, the limit of laminar behavior was determined by both the water concentration and the $N_{Re}$ number.

Yiğit [7] used numerical (Crank-Nicholson method; an implicit numerical solution method) to study heat and mass transfer in falling film absorber to solve the partial differential equations with given boundary conditions. Such equations were expressed in finite difference form in cylindrical coordinates. The coolant side flow rate (or, heat transfer coefficient) effects were found to be small on the values of mass absorbed, the outlet film temperature, and also the outlet mass fraction. Moreover, the inlet coolant temperature was found to influence the mass absorbed.

Kamei and Oishi [8], based on their experimental results concerning the rate of mass transfer in a falling liquid film, concluded that there exists four regions. A first region where $N_{Re} < 35$ for which the rate may be predicted by means of the equation valid for the laminar steady flow. A second region $35 < N_{Re} < 150$ for which the rate may be predicted if one replaces the diffusion coefficient $D$ found in the equation valid for the steady laminar case, by an apparent diffusion coefficient $D_p$ independent of $x$. A third region $150 < N_{Re} < 1000$ for which no correlation was possible. The fourth corresponds to $N_{Re} > 1000$ (turbulent regime).

The Maxwell–Stefan diffusion is a model for describing diffusion in multi-component systems. The basic assumption of the theory is that a deviation from equilibrium between the molecular friction and thermodynamic interactions leads to the diffusion flux. The molecular friction between two components is proportional to their relative velocities and their mole fractions. The gradient of chemical potential is the driving force of diffusion; nevertheless, for complex systems, such as electrolytic solutions, the equation must be expanded to incorporate additional terms for interactions other than chemical potential [9,10].

In the following study, an analytic, open-form (i.e., power series) solution is presented for a steady-state mass transfer of gas absorption into a falling liquid film under laminar condition. Furthermore, an overall mass transfer coefficient is defined in terms of an overall concentration difference. The objectives of the study are to determine the overall mass transfer coefficient which will enable the estimation of mass transfer rate and to compare the estimated mass transfer rate with that given by the convective mass transfer (bulk side mass transfer) as a means of model testing.

**MODEL DEVELOPMENT**

First, the momentum balance will be carried out to find the velocity profile. Consider a falling liquid film, as shown in Figure 1, flowing in the y-direction. The film has a thickness $\delta$ that extends from $z=0$ at the liquid/gas interface to $z=6$ at the solid/liquid interface. The film is infinitely wide with a width $W$ that extends in the page (i.e., x-direction). Such a thin liquid film is made of a mixture of volatile A and nonvolatile B, falling in laminar flow at steady state down one side of a vertical surface and exposed to gas, A, on the other side of the film, as shown in Figure 1. There is no bulk flow in the $z$-direction (i.e., the liquid velocity in the $z$-direction, $u_z$ is assumed zero.

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Momentum balance in y-direction

By solving the ordinary differential equation that describes the steady-state momentum balance in y-direction, found in many transport phenomena textbooks, the velocity in y-direction is expressed as:

$$u_y = \frac{2g\delta^2}{2\mu} \left[ 1 - \left( \frac{z}{\delta} \right)^{1/3} \right]$$

(1)

The mean flow velocity is given by:

$$\bar{u_y} = \int_{-\delta}^{\delta} u_y \, dz / \delta = \frac{\rho g \delta^2}{3\mu}$$

(2)

Let us define $\Gamma = \frac{\text{mass flow rate}}{\text{width}} = \frac{\text{mass flow rate}}{W}$

(3)

Equate Eq. (2) with (3):

$$\bar{u_y} = \frac{\rho g \delta^2 W}{3\mu} \Rightarrow \bar{u_y} = \frac{\Gamma}{\rho \delta}$$

(4)

Let $N_w = \frac{D_{ab} \bar{u_y} \rho}{\mu}$

(5)

$$D_{ab} = 4 \times \text{hydraulic radius} = 4r_w = 4 \times \frac{\text{cross-sectional area for flow}}{\text{wetted perimeter}} = 4 \times \frac{W\delta}{W} = 4\delta$$

(6)

$$N_w = \frac{D_{ab} \bar{u_y} \rho}{\mu} = \frac{4\delta \bar{u_y} \rho}{\mu} = \frac{4\Gamma}{\mu}$$

(7)

$$\begin{cases} N_w < 8 - 25 & \text{The flow is laminar; the film is flat} \\ 8 - 25 < N_w < 1200 & \text{The flow is laminar; the film is wavy} \end{cases}$$

Steady-state molar balance of solute A

Carrying out a steady-state solute balance, we reach at:

$$D_{ab} \frac{\partial^2 C_A}{\partial z^2} = \bar{u_y} \frac{\partial (C_A)}{\partial y}$$

(8)

B.C. #1: $\frac{\partial z = 0, C_A = C_{Ai}}$

(9a)

B.C. #2: $\frac{\partial z = \delta, \frac{\partial C_A}{\partial z} = 0}$

(9b)

B.C. #3: $\frac{y = 0, C_A = C_{Ai}}$

(9c)

Let us define the dimensionless concentration:

$$\theta = \frac{C_A - C_{Ai}}{C_{Ai} - C_{Ai}}$$

(10)

$$D_{ab} \frac{\partial^2 \theta}{\partial z^2} = \bar{u_y} \frac{\partial (\theta)}{\partial y}$$

(11)

$$z = 0, \theta = 0$$

(12a)

$$z = \delta, \frac{\partial \theta}{\partial z} = 0$$

(12b)

$$y = 0, \theta = 1$$

(12c)

Let $C^2 = \frac{D_{ab}}{u_y}$, Eq. (11) becomes:

$$\frac{\partial (\theta)}{\partial y} \frac{\partial \theta}{\partial z} \rightarrow \frac{\partial (\theta)}{\partial z} = C^2 \frac{\partial^2 \theta}{\partial z^2}$$

(13)

Assume: $\theta(z, y) = F(z) \times G(y)$

(14)

Carrying out partial differentiation once with respect to $y$ and twice with respect to $z$:

$$\frac{\partial \theta}{\partial y} = FG; \quad \frac{\partial^2 \theta}{\partial z^2} = GF^n$$

Substitute into Eq. (13) to get:

$$FG = C^2 GF^n$$

(15)

Rearrangement of Eq. (15) results in:

$$G^n \cdot \frac{F^n}{C^2} = \phi = \text{constant}$$

(16)

In general, $\phi$ may assume zero; negative; positive; or even a complex value. The physical problem at hand will determine the potential value of $\phi$.

Case 1:

$$\phi = 0$$

The trivial case

Case 2: $\phi > 0$

Solving for:

$$G^n \cdot \frac{F^n}{C^2} = \phi \Rightarrow G = C^2 e^{(\phi C^n)}$$

This will violate the physical condition that $\theta(z,y)$ decreases with increasing $y$. In fact, as $y$ increases, $C_A$ will approach $C_{Ai}$ (i.e. $\theta(z,y) \rightarrow 0$)

Case 3:
φ < 0

\[ \frac{G'}{C^2 G} - \frac{F''}{F} = \phi = -P^2 \]

Determine solutions of the ordinary differential equation (O.D.E.), which will satisfy the B.C.'s.

\[ \frac{F''}{F} = -P^2 \Rightarrow -P^2 F = F'' \Rightarrow F'' + P^2 F = 0 \]

Eq. (17) is considered a constant coefficient 2nd-order O.D.E. with a characteristic equation:

\[ (D^2 + P^2)F = 0 \Rightarrow D = \pm \sqrt{-P^2} = \pm Pi \]

The solution for Eq. (17) will be of the form:

\[ F(z) = A \cos(Pz) + B \sin(Pz) \]

Applying B.C. #1: @ \( z = 0, 0(0, y) = 0 \) for all \( y > 0 \), one gets:

\[ \theta(0, y) = 0 = F(0) \Rightarrow F(0) = 0 \Rightarrow A \cos(0) + B \sin(0) = 0 \Rightarrow A = 0 \]

\[ F(z) = B \sin(Pz) \]

Applying \( \Rightarrow B.C. \#2: @ z = \delta, \frac{\partial \theta}{\partial z} |_{z=\delta} = 0 \)

for all \( y > 0 \) \( \Rightarrow GF |_{z=\delta} = 0 \Rightarrow F' |_{z=\delta} = 0 \)

Keep in mind that \( G(y) \) for all \( y > 0 \) cannot be zero; else the solution will become trivial.

\[ F(z) = B \sin(Pz) \Rightarrow F'(z) = BP \cos(Pz) \Rightarrow BP \cos(Pz) |_{z=\delta} = 0 \]

This implies that \( BP \cos(P\delta) = 0 \)

Again, notice that neither \( P \) nor \( B \) can be set to zero; otherwise a trivial solution is obtained.

Thus, \( \cos(P\delta) = 0 \Rightarrow P\delta = (2n-1)\frac{\pi}{2} \)

(20)

Notice that Equation (20) gives us a formula for defining the parameter \( P \).

\[ P = \frac{(2n-1) \frac{\pi}{2}}{\delta} \]

Substituting \( P \) in Eq. (19), we get:

\[ F(z) = B_n \sin \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} z \right) \Rightarrow F(z) = B_n \sin \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} z \right) \]

\[ n = 1, 2, 3, \ldots \]

(22)

\[ \frac{G'}{C^2 G} = -P^2 \Rightarrow G = -C^2 P^2 G \Rightarrow G + C^2 P^2 G = 0 \]

Let us define \( \beta = C^2 P^2 \) which implies that: \( G + \beta G = 0 \) and the solution will be:

\[ G(y) = C_n e^{-\beta y} = C_n e^{-\left(\frac{(2n-1) \frac{\pi}{2}}{\delta} y \right)} \Rightarrow G(y) = C_n e^{-\left(\frac{(2n-1) \frac{\pi}{2}}{\delta} y \right)} \]

(23)

Since in the first step, it was assumed that \( \theta(z, y) = F(z) \times G(y) \)

then we have:

\[ \theta(z, y) = F(z) \times G(y) = B_n \sin \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} \right) \times C_n e^{-\left(\frac{(2n-1) \frac{\pi}{2}}{\delta} y \right)} \]

Combining the two constant as one constant such that \( b_n = B_n \times C_n \) then we have:

\[ \theta(z, y) = b_n \sin \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} \right) \times e^{-\left(\frac{(2n-1) \frac{\pi}{2}}{\delta} y \right)} \]

(24)

Based on the fundamental theorem, the linear combination of such \( n \) solutions will also be a solution for the original P.D.E.

Thus, \( \theta(z, y) = \sum_{n=0}^{\infty} b_n \sin \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} \right)x \times e^{-\left(\frac{(2n-1) \frac{\pi}{2}}{\delta} y \right)} \]

(25)

\( b_n \) must be chosen such that \( \theta(z, y) \) becomes equivalent to \( f(z) \) present in the half-range expansion Fourier series, namely Fourier sine series. Exploiting: \( @ y = 0, \theta(z, 0) = 1 \) for all \( z \), Eq. (25) becomes:

\[ \theta(z, 0) = 1 = \sum_{n=0}^{\infty} b_n \sin \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} \right)x \times e^{-\left(\frac{(2n-1) \frac{\pi}{2}}{\delta} y \right)} \]

(28)

The final solution is:

\[ C_n = C_{n+1} \]

\[ \frac{C_n - C_{n+1}}{C_{n+1} - C_{n}} = 4 \sum_{n=0}^{\infty} \sin \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} \right)x \times e^{-\left(\frac{(2n-1) \frac{\pi}{2}}{\delta} y \right)} \]

(29)

The flux of \( A, N \) can be evaluated at \( z=0 \).

\[ N_x = -\frac{\partial A}{\partial z} |_{z=0} = -D \frac{\partial A}{\partial z} |_{z=0} = 0 \]

(30)

\[ \frac{\partial A}{\partial z} |_{z=0} = \frac{4}{\delta} \sum_{n=0}^{\infty} \left[ \left( \frac{n}{2} \right) \sin \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} \right)x \times e^{-\left(\frac{(2n-1) \frac{\pi}{2}}{\delta} y \right)} \right] \]

(31)

\[ N_x = \frac{2D}{\delta} (C_n - C_{n+1}) \sum_{n=0}^{\infty} \left[ \frac{D_n}{\delta} \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} y \right) \right] \]

(32)

Equation (32) gives the local flux evaluated at a given value of \( y \). To find the total molar flow rate of \( A \), then we need to integrate this over \( dA \), that is, \( W_{dy} \).

\[ \bar{n}_A = \int_0^1 W_{dy} = \int_0^{\frac{2D}{\delta}} (C_n - C_{n+1}) \sum_{n=0}^{\infty} \left[ \frac{D_n}{\delta} \left( \frac{(2n-1) \frac{\pi}{2}}{\delta} y \right) \right] dy = \]

(33)
Based on the overall concentration difference, which is given by either Eq. (37)

\[ \eta > 0.1 \text{ for a value of } C_{Ai} \text{ value, which exist at the plane } z=0 \text{ and plane } z=\delta \text{ respectively.} \]

They also showed the concentration profile as a function of \( z \) and the initial \( C_{Ai} \) value, which are found at the plane \( z=0 \) and plane \( z=\delta \) respectively.

Let us introduce some dimensionless numbers

\[ N_{Sh} = \text{Schmidt number} = \frac{\mu / \rho}{D_{ab}} = \frac{\nu / \rho}{D_{ab}} \]

\[ N_{Pe} = \text{Peclet number for mass transfer} = \frac{4\delta \bar{\mu} / \rho}{D_{ab}} \]

Table 1: Summary of the oxygen absorption process into the falling water film.

<table>
<thead>
<tr>
<th>( \rho_{10 \text{ atm}} )</th>
<th>( T=25^\circ \text{C}=298 \text{K} )</th>
<th>( L=1.00 \text{ m} )</th>
<th>( W=6 \text{ cm} )</th>
<th>( N_A=50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A=Oxygen</td>
<td>B=Water</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_{a,b}=2.5 \times 10^{-5} \text{ cm}^2/\text{s} )</td>
<td>( \nu_{10}=2.3 \times 10^{-4} )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Eq. (37) while retaining the first two terms in the power series:

\[ \eta = \frac{2D_{ab}L}{3N_{Sh}^2} \]

They also showed the concentration profile as a function of \( \eta \) given by:

\[ \ln \left( \frac{C_{Ai} - C_{A0}}{C_{Ai} - C_{A0}} \right) = -\left[ \ln(0.7857) - 5.1213\eta \right] = (0.241 + 5.1213\eta) \]

Finally, it should be noticed that the molar flow rate of absorption of \( A \), \( \dot{n}_A \), from the gas into the liquid film for height \( L \) and width \( W \) may also be given by the difference between the outlet and inlet bulk values of flow rate of \( A \):

\[ \dot{n}_A = \pi \delta W \left( C_{A0} - C_{Ai} \right) \]

RESULTS AND DISCUSSION

For the sake of testing the proposed model for evaluating the mass transfer coefficient, \( \bar{k}_c \), based on the overall concentration difference, that is, between the interface \( (C_{Ai}) \) value and the initial \( (C_{Ai}) \) value, which exist at the plane \( z=0 \) and plane \( z=\delta \) respectively.

Example:

Let us estimate the rate of absorption of pure oxygen at 10 atm and 25°C into water flowing as a film down a vertical wall 1 m high and 6 cm in width at a Reynolds number of 50 without surface ripples. Assume the diffusivity of oxygen in water is 2.5 \( \times 10^{-5} \text{ cm}^2/\text{s} \) and that the mole fraction of oxygen in water at saturation for the above temperature and pressure is 2.3 \( \times 10^{-4} \).
\[
\bar{C}_{\text{at}} = \frac{C_{a} - C_{w}}{C_{w}} = 4 \pi \left[ \left( \frac{1}{2} \sin \left( \frac{\pi}{2} \theta \right) \right) \pm \frac{1}{2} \delta \right]
\]

(53)

\[
\bar{C}_{\text{at}} = \frac{C_{a} - C_{w}}{C_{w}} = 4 \pi \left[ \left( \frac{1}{2} \sin \left( \frac{3 \pi}{2} \theta \right) \right) \pm \frac{1}{2} \delta \right]
\]

(54)

\[
\bar{C}_{\text{at}} = 0.012501 \text{ mol L}^{-1}
\]

(55)

From Eq. (46):

\[
\dot{n}_a = \bar{C}_{\text{at}} \times \delta \times W \times \left( C_{a} - C_{w} \right)
\]

\[
\dot{n}_a = 0.0773 \text{ mol s}^{-1} \times 0.000146 \text{ m} \times 0.06 \text{ m} \times 8.153 \times 10^{-4} \text{ mol L}^{-1} = 0.815 \times 10^{-5} \text{ mol s}^{-1}
\]

(56)

From Eq. (36):

\[
\dot{n}_a = \frac{C_{a} - C_{w}}{L} \times \beta \times W
\]

\[
\dot{n}_a = 0.9946 \times 10^{-5} \text{ mol s}^{-1} \times 0.0012763 \times 4.6 \times 10^{-4} \text{ mol L}^{-1} = 0.815 \times 10^{-5} \text{ mol s}^{-1}
\]

(57)

Percent Relative Error (PRE) =

\[
\frac{\text{True} - \text{Estimated}}{\text{True}} \times 100\% = \frac{0.815 \times 10^{-4} \text{ mol L}^{-1} - 0.734 \times 10^{-5} \text{ mol L}^{-1}}{0.815 \times 10^{-4} \text{ mol L}^{-1}} = 9.9\%
\]

(58)

Seader and Henley [11] quoted: "Although this is the most widely used approach for defining a mass-transfer coefficient, in this case of a falling film it fails because $\frac{dC}{dz}$ at $z = 0$ is not defined. Therefore, for this case we use another approach as follows ...". In their context, the phrase "is not defined" means that there is no analytic (closed- or open-form) solution available to estimate the flux (i.e., the gradient) at $z = 0$.

With this approach, the derivative can now be defined and is given by Eq. (30) as an analytic, open-form solution.

Let us also compare our solution by that of Seader and Henley [11].
CONCLUSION

The overall mass transfer coefficient \( \mathbf{K} \), for a binary diffusion system, was evaluated in light of the analytic, open-form solution and was tested for a gas absorption case. The amount of absorbed \( A \) given by the mass transfer coefficient being multiplied by the area available for mass transfer and by the overall concentration difference, \( \mathbf{n} = \mathbf{K} (C_{in} - C_{ol})WL \), is essentially the same as that given by the bulk terms on the liquid side, \( \dot{n} = \mathbf{D} \delta (C_{in} - C_{ol}) \). The bulk transport rate of gas solute was compared with that given by Seader and Henley [11] and the results were in agreement.

REFERENCES