Short Communication

LDL Transport in Blood Vessels using Kinetic Theory

Dimitri Gidaspow*
Department of Chemical and Biological Engineering, Illinois Institute of Technology, USA

Abstract

Unequal granular temperature theory predictions agree with measurements of RBC and platelets in blood vessels. The effect of shear on LDL transport is predicted only qualitatively.

REVIEW OF RBC AND PLATELET TRANSPORT

Conventional transport phenomena [1-4] are unable to explain the formation of plaque in blood vessels in the region of low shear, such as in the carotid bifurcation [5]. Recently Gidaspow and [6] used the unequal granular temperature kinetic theory to compute the migration of red blood cells (RBC) away from the vessel walls and the platelets to the walls, in developed flow. The approximate analytical solution for the platelet concentration, \( n \) is given by the expression

\[
n = \frac{n_{\text{inlet}}}{1 - (r/R)^4}
\]

(1)

Where \( R \) is the tube radius and \( r \) is the radial coordinate. This expression agreed well with the [7] experimental data. The RBC distribution and its granular temperature were computed using essentially the same theory as that of [8]. Figure 1 shows a comparison of the granular temperatures computed using the fully developed flow boundary value problem to that of the commercial code, AnsysFluent. As stated in the [8] paper the large fluid-particle interaction term in the FLUENT granular temperature equation was deleted, with the help of [9] of the US Department of Energy, NETL who programmed such equations into their MFIX code. With this change Ansys Fluent should allow correct simulation of the flow and migration of the red blood cells away from the walls for the carotid artery, as was done for pulsative flow in a coronary artery without branches by [10]. The more realistic case with branches was recently done for the single phase flow situation in which there is no clear layer near the walls.

For the kinetic theory simulations our CFD code is fully described in the 2009 book by [11]. The FORTRAN code is connected to MATLAB 5.3 for data analysis and movies.

Figure 2 shows a comparison of the computed RBC distributions to [12] experiment. The RBC distributions were obtained from a theoretical equation of state.

Our computed RBC distributions agreed only qualitatively with [7] experiment, probably due to the need to include cohesion in the equation of state.

LDL TRANSPORT

The simplest way to obtain the LDL distribution in blood
vessels is to relate them to RBC distributions. In the kinetic theory of gases the temperatures of components are equal because molecules do not lose energy upon collision. With the usual definition of granular temperature as the average variance of particle velocities, \( (m/s)^2 \) the rough approximation of no energy dissipation gives the equation,

\[
\text{mass}_{\text{RBC}} \theta_{\text{RBC}} = \text{mass}_{\text{LDL}} \theta_{\text{LDL}}
\]  

(2)

Where \( \theta \) is the granular temperature. Hence the granular temperature of LDL can be estimated from the RBC granular temperature, shown in (Figure 1). It is high next to the wall and low at the center. The LDL pressure can be calculated from its equation of state, ideal particle law, as shown below:

\[
P_{\text{LDL}} = \varepsilon_{\text{LDL}} P_{\text{LDL}} \theta_{\text{LDL}}
\]  

(3)

Where \( P \) is the pressure, \( \varepsilon \) is the volume fraction and \( \rho \) is the density. The pressure can be estimated from inlet conditions. Then the volume fraction of LDL can be obtained from equation 3. Like in (Figure 2), the concentration of LDL will be low at the artery walls. A low shear will give a low granular temperature and hence a high wall LDL concentration.

Unfortunately the assumption of no kinetic energy dissipation is far off, because Equation 2 gives an unreasonable high granular temperature for LDL.

An alternate approximation is to neglect conduction, but to keep dissipation with an average restitution coefficient, \( e \). Then a balance between dissipation and production of oscillations gives the equation [13]:

\[
(1-e) \theta = \frac{1}{15} \left( \frac{\partial u}{\partial y} \right)^2 \rho
\]  

(4)

Equation 4 shows that a low shear rate, \( \partial u / \partial y \) gives a low granular temperature, \( \theta \) and hence a high LDL wall concentration, according to Equation 3. Unfortunately we do not know the loss of energy, given by \( e \). Rotation of RBC has also been included into the theory with their own restitution coefficients [14].

Transport of LDL and HDL is also affected by surface charge. The zeta potentials of HDL and LDL are -10.5 and -4.5 mV, respectively. These differences result in different granular pressures of LDL and HDL and therefore different concentration profiles for LDL and HDL.

But it appears that a quantitative prediction of LDL and HDL nanoparticle distributions is not possible at this time without measurements.

REFERENCES