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Review Article

Sachs -Wolfe

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Abstract

Using the scalarly perturbed FLRW metric we investigate the effect of gravitational redshift on the CMB photons temperature. Having done this we investigated how the gravitational time di- lation due to the scalar perturbation in the metric affects the variation of this same temperature. Finally, we infer the result corresponding to the composition of the two effects in macroscopic formulation: the Sachs-Wolfe Effect (SWE). Having done this, we proceed to the analysis of this same phenomenology this time via a kinetic approach. Through this channel, in addition to the SWE, we detect the action of the Integrated Sachs-Wolfe Effect (ISW) deforming the CMB power spectrum.

INTRODUCTION

In standard cosmology, **Einstein's Field Equations** (EFE) were established as rulers that describe the dynamic evolution of the Universe [1]. Considering the **cosmological principle** the metric (solution for the EFE) that best fits the description of this dynamic evolution is the FLRW [2-5].

Over the EE a so-called **standard cosmological model** (ΛCDM) was formulated. This model is the one that best fits observational data obtained from new scientific ventures, such as the Planck project *a.e.* among others. However, this model faces some difficulties in theoretical justification for some observational notes that have so far been unresolved. Among these difficulties is the search for a phenomenological justification for anisotropies in the CMB power spectrum. One of these anisotropies corresponds to the so-called **SACHS- WOLFE EFFECT** (SWE), subject of this work.

The SWE [6], arises from the gravitational *redshift* suffered by the CMB during the **Last Scatering Surface** (LSS) combined with the effect (*blueshift*) of time dilation due to some scalar disturbance in the metric also when LSS. This effect is not constant across the sky due to differences in matter and energy densities during LSS [7-11].

- CMB ANISOTROPIES -

The **correlation function** (first definition) between two points, in a space with spherical symmetry, can be decomposed in the form (second definition)

$$C(\vartheta) \equiv \left\langle \frac{\Delta T}{T}(\hat{n}_1) \frac{\Delta T}{T}(\hat{n}_2) \right\rangle \equiv$$

$$\equiv \frac{1}{4\pi} \sum_{l=2}^{\infty} (2\ell+1) c_\ell^2 P_\ell(\cos\vartheta) \quad (1)$$

where the relationship between ϑ and ℓ

$(\cos\vartheta \equiv \hat{n}_1 . \hat{n}_2)$ is given by

$$\vartheta \equiv \frac{180^\circ}{\ell} \; . \quad (2)$$

It is customary to adopt as **Temperature fluctuations** ($y-$ axis) the function

$$(\Delta T)^2 \equiv \left[\frac{\ell(\ell+1)}{2\pi} c_\ell \right] \langle T \rangle^2$$

for large ℓ $P_\ell(\cos\vartheta) \simeq P_\ell(1) = 1$ and the $c\ell s$ are defined in terms of the observational data for large ℓ.

The monopole term (ℓ = 0) is the constant isotropic mean temperature of the CMB ⟨T ⟩ = 2.7255 ± 0, 0006K with one standard deviation confidence. This term must be measured with absolute temperature devices such as the FIRAS instrument on the COBE satellite.

The CMB dipole represents the largest anisotropy which is in the first spherical harmonic (ℓ = 1) a cosine function.

$$\sqrt{c_1} \simeq 3.10^{-3} \quad (3)$$

$$\sqrt{c_2} = 0,76 \pm 0,24.10^{-5} \text{ (COBE)} \quad (4)$$

For small ones ℓs [12],

$$c_\ell \simeq \frac{6c_2}{\ell(\ell+1)} \qquad (5)$$

It is customary not to represent these first two terms in the CMB power spectrum.

The SW and ISW occur on large scales (small ℓs).

FIRST ACOUSTIC PEAK: LSS

It is expected that there will be a peak in temperature variation in the photon radiation (from the CMB) in the LSS region due to the difference in pressure before and after the last scattering [13-19].

$$a(t) = \left(\frac{t}{t_r}\right)^{1/2} \quad \text{com} \quad t_r = 3,1017.10^{19}\text{s}\,; \qquad (6)$$

the physical dimension of the causal horizon in the LSS was worth

$$(\Delta H)_{\text{LSS}} = a\int \frac{cdt}{a} = 3ct_r(1+z_{\text{LSS}})^{-3/2} \simeq 252\,\text{kPc} \quad (7)$$

However, the horizon that really matters is associated with the distance traveled by sound: the behavior of pressure oscillations.

It is known that the **speed of sound** at that time was

$$c_s \simeq \frac{\sqrt{3}}{3}c \;\rightarrow\; (\Delta H)_s \simeq 145\,\text{kPc} < (\Delta H)_{\text{LSS}}\,. \qquad (8)$$

From the expression that converts distance into horizon angle in the spatially flat Universe model where h is the **Planck constant**

$$\vartheta_{\text{LSS}} = 0,57'(\Delta H)_s h \simeq 0,99^\circ\,. \qquad (9)$$

To this ϑ corresponds $\ell \simeq 182$: : approximate position of the first acoustic peak of Figure 1.

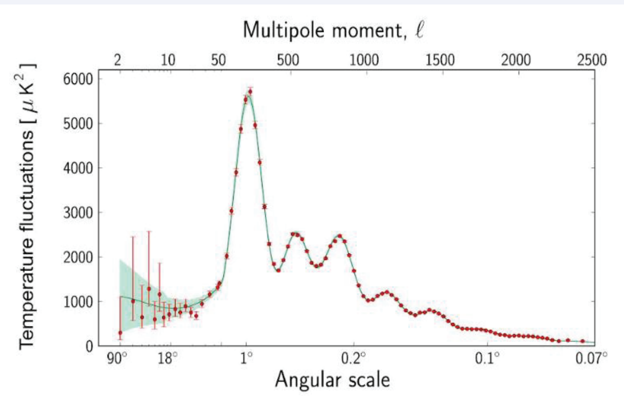

Figure 1 Power Spectrum (PS) of the **Cosmic Microwave Background** (CMB).

MACROSCOPIC SACHS-WOLFE EFFECT

Consider the **scalarly perturbed flat FLRW metric (conformal Newtonian gauge)** in the form

$$ds^2 \equiv a^2\left[\left(1+\frac{2\Phi}{c_s^2}\right)d\eta - \left(1-\frac{2\Phi}{c_s^2}\right)\delta_{ij}\,dx^i dx^j\right]\,. \quad (10)$$

In this context, Φ corresponds to the **Newtonian potential** itself.

The Gravitational "Redshift"

Establishing the velocity of the fluid by $\mathbf{V} \equiv \mathbf{V}(t)$ the Euler equation for a fluid with **Total** (T) internal energy $\rho_{(T)}$ becomes [11-24]

$$\frac{\partial(\delta\mathbf{V})}{\partial t} = -(\mathbf{V}.\nabla)\mathbf{V} - \frac{1}{\rho_{(T)}}\nabla(\delta P) - \nabla(\delta\Phi)\,, \quad (11)$$

where $\nabla(\delta\Phi)$ corresponds to the action of some external force δ**F.** If the only external force acting on the system is gravitational, we can identify the Φ of gravity with that of gravitational (10).

Establishing $\rho \equiv \rho(x^\mu)$, as each **degree of freedom** has associated with it the amount of energy $\rho_{(0)} = k_B T/2$, in four dimensions we have $\rho_{(T)} = 4\rho_{(0)}$.

$$\frac{\partial(\delta\mathbf{V})}{\partial t} = -\frac{1}{4\rho_{(0)}}\nabla\left[\left(\frac{dP}{d\rho}\right)\delta\rho\right] - \nabla(\delta\Phi) = $$
$$= -\frac{c_s^2}{4\rho_{(0)}}\nabla(\delta\rho) - \nabla(\delta\Phi)\,, \qquad (12)$$

where c_s represents the EoS parameter $P \equiv c_s^2\rho$.

As from **CMB blackbody spectrum**

$$\rho \propto T^4$$
$$\Rightarrow\; \delta\rho = 4T^3\delta T \;\Rightarrow\; \frac{\delta\rho}{\rho_{(0)}} \propto 4\frac{T^3}{T^4}\delta T$$
$$\Rightarrow\; \nabla(\delta\rho) = 4\rho_{(0)}\nabla\Theta \qquad (13)$$

Where

$$\Theta \equiv \frac{\delta T}{T}\,.$$

Replacing (13) in (12)

$$\frac{\partial(\delta\mathbf{V})}{\partial t} = -c_s^2\nabla\Theta - \nabla(\delta\Phi)\,. \qquad (14)$$

From the **continuity equation**,

$$\frac{\partial}{\partial t}(\delta n) + \nabla(n\,\delta\mathbf{V}) = 0 \;\Rightarrow\; \nabla(\delta V) = -\frac{\delta\dot{n}}{n}\,. \qquad (15)$$

Again from the CMB spectrum

$$n \propto T^3 \;\Rightarrow\; \frac{\delta\dot{n}}{n} = 3\dot{\Theta}\,. \qquad (16)$$

In these terms, (15) becomes

$$\nabla(\delta V) = -3\dot{\Theta}\,. \qquad (17)$$

Replacing the previous one in ∇ of (14)

$$\ddot{\Theta} - \frac{1}{3}[c_s^2\,\nabla^2\Theta + \nabla^2(\delta\Phi)] = 0\,. \qquad (18)$$

In terms of the Inverse Fourier Transforms of Θ and δΦ, we arrive at

$$\ddot{\Theta}_k + \frac{c_s^2k^2}{3}\Theta_k + \frac{k^2}{3}\delta\Phi_k = 0 \qquad (19)$$

In relativistic analysis we must consider the action of gravitational *redshift* on the law of temperature evolution. Such action can be modeled according to the addition *ad hoc*, in (19), of some term such that it cancels $\ddot{\Theta}_k$ when added to it.

$$\delta\Phi_k = -c_s^2\Theta_k \;\Rightarrow\; \delta\ddot{\Phi}_k = -c_s^2\ddot{\Theta}_k\,. \qquad (20)$$

This will be the term to be added to (19): $\ddot{\Theta}_k$.

$$\Rightarrow \not{\ddot{\Theta}}_k + \frac{c_s^2k^2}{3}\Theta_k + \frac{k^2}{3}\delta\Phi_k + \frac{\not{\delta\ddot{\Phi}_k}}{\not{\omega}} = 0$$

Thus, the term $\delta\Phi/\omega$ corresponds to a variation in temperature due to the gravitational *redshift* suffered by the CMB

$$\left(\frac{\delta T}{T}\right)_{redshift} = -\frac{\delta\Phi}{c_s^2}\,. \qquad (21)$$

A Dilatation (Gravitational) of Time

Consider, in the matter era, $a(t) \propto t^{2/3}$ and $T \propto 1/a$.

$$\Rightarrow \frac{\delta T}{T} = -\frac{1/a^2}{1/a}\delta a = -\frac{\delta a}{a}\,.$$

On the other hand.

$$\frac{\delta a}{a} = \frac{2}{3}\frac{\delta t}{t}\,.$$

From the relation for **gravitational time dilation** $t = t_0/\sqrt{g_{00}}$ where $g_{00} = 1 + 2\Phi/c_s^2$; expanding $1/\sqrt{g_{00}}$ in Taylor series and substituting in the relation for t,

$$\frac{\delta t}{t_0} \simeq -\frac{\delta\Phi}{c_s^2}\,. \qquad (22)$$

$$\Rightarrow \left(\frac{\delta T}{T}\right)_{dilation} = \frac{2}{3}\frac{\delta\Phi}{c_s^2}\,. \qquad (23)$$

Interms of (21) and (23), we can estimate a total temperature deviation: the SWE

$$\left(\frac{\delta T}{T}\right)_{SW} \equiv -\frac{\delta\Phi}{c_s^2} + \frac{2}{3}\frac{\delta\Phi}{c_s^2} \simeq -\frac{1}{3}\frac{\delta\Phi}{c_s^2} \qquad (24)$$

This last result corresponds to the well-known SWE: CMB photons being *redshifted* (losing energy) as if they were climbing some potential barrier.

MICROSCOPIC TEMPERATURE FLUCTUATIONS

If there are scalar fluctuations in the metric, what is the effect of these fluctuations on the CMB spectrum?

Let the **metric FLRW** spatially "**flat**" (κ = 0) and scalarly perturbed

$$\Rightarrow ds^2 = \left[1 + 2\Phi(t,x^i)\right]dt^2 +$$

$$-a^2(t)\left[1 - 2\Psi(t,x^i)\right]\delta_{ij}dx^idx^j\,, \qquad (25)$$

where "Φ" represents the **gravitational potential** at the **Newtonian** limit and "Ψ" a **change in the metric due to the curvature of space**, it comes to

$$P^0 \equiv \frac{(\sum(P_i)^{1/2})}{a^2} \equiv \frac{p}{a^2} \qquad \text{e} \qquad P_0 = (1+2\Phi)\,p\,.$$

The relation that describes the temporal evolution of the Boltzmann distribution function on the "*mass shell*" $f(t, x^j, P^j)$ is called the **Boltzmann Equation** (BE)

$$\frac{d}{dt}f\left(t,x^j,P^j\right) = \mathcal{C}(f)\;; \qquad (26)$$

where there corresponds to $C(f)$ some kinetic term to be adjusted according to the adopted cosmological model.

$$\frac{\partial f}{\partial t} + \frac{dx^i}{dt}\frac{\partial f}{\partial x^i} + \frac{dp}{dt}\frac{\partial f}{\partial p} = \mathcal{C}(f)\,, \qquad (27)$$

where, starting from $p^2 \equiv -g_{ij}P^iP^j$

$$\Rightarrow\; (1+2\Phi)(P^0)^2 - p^2 = 0$$

$$\therefore\; P^0 = \frac{1}{\sqrt{1+2\Phi}}\,p \qquad (28)$$

we expanded the previous relation in a power series up to the 1st order in Φ

$$P^0 = \frac{1}{\sqrt{1+2\Phi}}\,p \simeq (1-\Phi)\,p \quad \therefore\; P^0 \simeq (1-\Phi)\,p \quad (29)$$

and then we substituted it in

$$\frac{dx^i}{dt} = \frac{dx^i}{d\lambda}\frac{d\lambda}{dt} = \frac{P^i}{P^0} \simeq \frac{(1+\Psi)}{(1-\Phi)}\frac{\hat{n}^i}{a} \simeq (1+\Phi+\Psi)\frac{\hat{n}^i}{a}\,. \qquad (30)$$

Finally, considering only zero-order terms, by implementing this we reach the second term on the left of the approximation (27).

Let us now calculate the term dp/dt of (27).

Starting from **geodetic equation** in terms of the same arbitrary affine parameter λ under the

$$\Gamma^{\mu}_{\lambda\kappa} = g^{\mu\nu}\left(\frac{\partial g_{\lambda\nu}}{\partial x^{\kappa}} - \frac{1}{2}\frac{\partial g_{\lambda\kappa}}{\partial x^{\nu}}\right),$$

doing $\mu = 0$

$$\Rightarrow \frac{dP^0}{dt} = \frac{g^{0\nu}}{2}\frac{\partial g_{\lambda\kappa}}{\partial x^{\nu}}\frac{P^{\lambda}P^{\kappa}}{P^0} - g^{0\nu}\frac{\partial g_{\nu\lambda}}{\partial x^{\kappa}}\frac{P^{\lambda}P^{\kappa}}{P^0}. \quad (31)$$

Dispensing terms from the orders Φ^2, Ψ^2 and $\Phi \times \Psi$,

$$\frac{dP^0}{dt} = -Hp - \frac{\partial \Phi}{\partial t}p + \frac{\partial \Psi}{\partial t}p - 2\frac{\partial \Phi}{\partial x^i}\frac{p}{a}\hat{n}^i. \quad (32)$$

On the other hand,

$$\frac{dP^0}{dt} = \frac{d}{dt}[(1-\Phi)p] = (1-\Phi)\frac{dp}{dt} - \frac{d\Phi}{dt}p =$$

$$\simeq \frac{dp}{dt} - \frac{\partial \Phi}{\partial t}p - \frac{\partial \Phi}{\partial x^i}\frac{p}{a}\hat{n}^i \quad (33)$$

in which the relationship to dxi/dt was used.

Equaling this to the derivative of (29) and taking the result in (27) considering f independent of x^i (isotropy),

$$\frac{\partial f}{\partial t} + \left(-H + \frac{\partial \Psi}{\partial t} - \frac{\partial \Phi}{\partial x^i}\frac{\hat{n}^i}{a}\right)p\frac{\partial f}{\partial p} = \mathcal{C}(f) \quad (34)$$

In this configuration

- H corresponds to the loss of energy due to the expansion of the Universe (zero-order term);
- $+\partial\Psi/\partial t$ corresponds, in terms of the metric described in form (25), to the "deviation" in energy due to the temporal variation of the perturbative term Ψ;
- $-(\partial\Phi/\partial x^i)(n^i/a)$, finally, corresponds to the spatial variation of gravity due to variations in matter density: photons "lose" energy "climbing potential (gravitational)" barriers"; and "gain" energy by "falling" into potential wells.

The Disturbance In T

For a relativistic gas of bosons (photons are bosons) in thermodynamic equilibrium, the distribution function $f_{(0)}$ takes the form

$$\boxed{f_{(0)} = \frac{2}{e^{(p/T_0)} - 1}}. \quad (35)$$

Consider, from

$$\frac{1}{T} \approx \frac{1}{T_0} - \frac{1}{T_0^2}(T - T_0) \approx \frac{1}{T_0}\left(1 - \frac{\delta T}{T_0}\right)$$

by construction, the **temperature fluctuation function**

$$\Theta \equiv \frac{\delta T}{T_0} = \frac{\delta T}{T_0}(t, x^i, \hat{n}^i). \quad (36)$$

Thus, in terms of Θ, the function $f_{(0)}$ can be described in approximate form

$$\Rightarrow f \simeq \frac{2}{e^{(p/[T(t)(1+\Theta)])} - 1}. \quad (37)$$

As it is known that $\delta T/T_0 \simeq 10^{-5}$, the previous ratio, expanded in a series of powers around T_0, can be described by

$$\Rightarrow f_{(0)} \simeq f_{(0)} + T\frac{\partial f_{(0)}}{\partial T}\Theta, \quad (38)$$

where $f_{(0)}$ is the **Bose-Einstein distribution function** (35).

Taking into account, also, that in zero order,

$$T\frac{\partial f_{(0)}}{\partial T} \simeq -p\frac{\partial f_{(0)}}{\partial p},$$

then, the (38) becomes

$$\therefore\ f \simeq f_{(0)} - p\frac{\partial f_{(0)}}{\partial p}\Theta. \quad (39)$$

Substituting the previous relation in the BE in terms of the parameter **conformal time** η

$$\left(\dot{\Theta} + \frac{\partial \Theta}{\partial x^i}\hat{n}^i - \dot{\Psi} + \frac{\partial \Phi}{\partial x^i}\hat{n}^i\right)p\frac{\partial f_{(0)}}{\partial p} = -a\mathcal{C}(f_{(0)}). \quad (40)$$

Decoupling Matter-Radiation: *$C(F_{(0)}) = 0$:* Immediately after the recombination phase, the uncoupled photons of matter start to propagate with an average free path of the order of + ∞, *i.e.*, without interacting with anything else. In terms of the perturbed BE, this condition corresponds to fixing $C(f_{(0)}) = 0$.

In these terms, the linearly perturbed BE (40) implies

$$\dot{\Theta} + \frac{\partial \Theta}{\partial x^i}\hat{n}^i = \dot{\Psi} - \frac{\partial \Phi}{\partial x^i}\hat{n}^i. \quad (41)$$

Adopting this convention, the inverse Fourier transform (see **appendix** A) of the previous one is

$$\Rightarrow \dot{\hat{\Theta}} + ik\mu\hat{\Theta} = \dot{\hat{\Psi}} - ik\mu\hat{\Phi}, \quad (42)$$

where $\mu \equiv \vec{k}.\hat{n}^i/k$ was defined as the cosine of the angle between the wave vector k and the direction of propagation of the photon $\hat{n}^i$..

Properly rearranged the Boltzmann equation can be described in the form

$$e^{-ik\mu\eta}\frac{\partial}{\partial \eta}\left(\hat{\Theta}e^{ik\mu\eta}\right) = \dot{\hat{\Psi}} - ik\mu\hat{\Phi}. \quad (43)$$

So

$$\therefore\ \frac{\partial}{\partial \eta}\left(\hat{\Theta}e^{ik\mu\eta}\right) + \frac{\partial}{\partial \eta}\left(\hat{\Phi}e^{ik\mu\eta}\right) = \frac{\partial}{\partial \eta}\left(\hat{\Phi} + \hat{\Psi}\right)e^{ik\mu\eta} \quad (44)$$

- SWE & ISW -

Integrating the previous relation into the period from recombination (η_r) to the present tense (η_0)

$$\Rightarrow \hat{\Theta}(\eta_0) + \hat{\Phi}(\eta_0) = \underbrace{\left[\hat{\Theta}(\eta_r) + \hat{\Phi}(\eta_r)\right] e^{ik\mu(\eta_r - \eta_0)}}_{\text{SW}} +$$

$$\underbrace{+ e^{-ik\mu\eta_0} \int_{\eta_r}^{\eta_0} \left(\dot{\hat{\Phi}} + \dot{\hat{\Psi}}\right) e^{ik\mu\eta}\, d\eta}_{\text{ISW}} \qquad (45)$$

The exponential factor in the previous [equation] represents the phase shift in the photon wavefronts; this shift should be disregarded in the present approach.

The equation above represents anisotropies in the CMB temperature spectrum observed today in terms of the anisotropies at the time of recombination.

In very dense regions and under the Newtonian gauge $\Psi \equiv \Phi > 0$, justifying the observed temperature fluctuation being $(\delta T/T_0) > 0$. This corresponds to the photon being redshifted (losing energy:"cooling") as if it were climbing some potential barrier.

Note that, disregarding the term ISW, the expected result is recovered in the 1st order of Φ:

$$\Rightarrow \Theta(\eta_0) + \Phi(\eta_0) = \Theta(\eta_r) + \Phi(\eta_r) = \text{constant}\,. \qquad (46)$$

This effect corresponds to the **SACHS-WOLFE EFFECT** (SWE) in its simplified form.

In these exact same terms it can be seen, from the previous relationship, that the temporal variation of Φ in the period that goes from t_r to t_0 also modifies the temperature spectrum of the CMB. This type of anisotropy corresponding to the spectrum of the CMB is called **INTEGRATED SACHS-WOLFE EFFECT** (ISW).

CONCLUSION

Starting from the Euler equation describing a gravitationally disturbed fluid of photons, we arrive, having established the blackbody conditions on the photons, whereby they undergo *redshift* due to the Φ perturbation during the LSS. The time dilation resulting from this same disturbance increases the temperature over time (*blueshift*). The resulting effect of the composition of these two processes (*redshift* in LSS plus time dilation) on the temperature spectrum of the CMB points to the photons cooling over time: the SWE.

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