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Review Article

The Universal Refined Equation (CUP-Ω*): Covariant GKLS Dynamics, Tomonaga–Schwinger Integrability, and Einstein–Langevin Coupling in the CUCE/Spinoza/Hilbert Framework

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Abstract

We present a covariant, completely positive and thermodynamically consistent refinement of the universal equation within the CUCE/Spinoza/Hilbert framework, denoted CUP-Ω*. The formulation unifies Tomonaga–Schwinger evolution on Cauchy hypersurfaces with a modular Gorini–Kossakowski–Lindblad–Sudarshan (GKLS) generator that obeys detailed balance in the GNS metric with respect to a unified thermodynamic target σ*. We prove (i) foliation independence under local commutation, (ii) complete positivity of the finite-step propagator, (iii) existence of a global Lyapunov functional $\Phi[\rho] = \int_\Sigma D_{rel}(\rho \| \sigma^*)$ ensuring the second law, (iv) primitivity with a unique attractor σ*, and (v) local conservation via consistent coupling to the Einstein–Langevin equation with conserved stochastic sources. We further outline falsifiable predictions with quantitative protocols in superconducting circuits and optomechanics.

INTRODUCTION

Spinoza's monism can be operationally reformulated in Hilbert-space language: one substance, many modes as observable algebras. Within this CUCE/Spinoza/Hilbert programme, the CUP-Ω* equation supplies a universal dynamical law that is (a) covariant at the level of foliation by Cauchy hypersurfaces, (b) completely positive at finite steps, and (c) thermodynamically consistent via a modular target $\sigma^\star$ capturing both KMS equilibrium and observer/prior information through an affine geometric mean. Our presentation emphasizes rigorous mathematical structure and empirical consequences.

MAIN EQUATION

Let ρ[Σ] be the state functional on a Cauchy hypersurface Σ. The local Tomonaga–Schwinger (TS) evolution at

x ∈ Σ reads

$$\frac{\delta\rho[\Sigma]}{\delta\Sigma(x)} = -i\,[H_\perp(x) + H_{\rm LS}(x),\,\rho] + \int d\tau\,\chi_\ell(\tau)\sum_\alpha \gamma_\alpha(x)\Big(L_\alpha\,\rho\,L_\alpha^\dagger - \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha,\rho\}\Big). \qquad (1)$$

The jump operators are modular with respect to the unified thermodynamic target $\sigma^\star$,

$$L_\alpha(x,\tau) = \sigma_\star^{1/2}(x)\,F_\alpha(x,\tau)\,\sigma_\star^{-1/2}(x), \qquad \gamma_\alpha(x)\geq 0, \quad \chi_\ell(\tau)\geq 0, \quad \int \chi_\ell(\tau)\,d\tau = 1, \qquad (2)$$

and

$$\sigma^\star(x) = \sigma_{\rm KMS}(x)\ {}_\eta\ \sigma_{O,P}(x), \qquad 0\leq\eta\leq 1, \qquad (3)$$

Where $A\#_\eta B$ denotes the affine geometric mean. Equation (1) couples to semiclassical gravity via a conserved stochastic source through

$$G_{\mu\nu}[g] + \Lambda g_{\mu\nu} + \hbar\langle\Delta G_{\mu\nu}\rangle_\rho = 8\pi G(\langle T_{\mu\nu}\rangle_\rho + \xi_{\mu\nu}), \qquad \nabla_\mu\langle T^\mu{}_\nu\rangle_\rho = \nabla_\mu \xi^\mu{}_\nu = 0. \tag{4}$$

Axioms

(Causal) *Local commutation*. For spacelike separated *x, y*, the local superoperators commute: $[\mathcal{L}(x), \mathcal{L}(y)] = 0$ and $[H_\perp(x), \mathcal{L}(y)] = 0$.

(CPTP) *Bochner positivity*. $\chi_\ell \geq 0$ and the rates $\{\gamma_\alpha\}$ arise from positive-definite (Bochner) environment correlators so that e $e^{\Delta\Sigma\,\mathcal{L}}$ is CPTP for any finite step $\Delta\Sigma$.

(GNS) *Detailed balance*. With modular jumps (2), the generator is symmetric in the GNS inner product $\langle A, B\rangle_{\sigma^\star} = \mathrm{Tr}(\sigma_\star^{1/2} A^\dagger \sigma_\star^{1/2} B)$.

(Prim) *Primitivity*. The set $\{F_\alpha\}$ generates the full local *-algebra (all $|i\rangle\langle j|$ in the spectral basis of $\sigma^\star$), implying a unique faithful stationary state.

(Cons) *Conservation and gauge*. F_α are BRST-invariant; (4) uses conserved noise with fluctuation–dissipation relations.

Global Lyapunov functional and the second law

$$\Phi[\rho] = \int_\Sigma D_{\mathrm{rel}}(\rho\|\sigma^\star)\, d\Sigma = \int_\Sigma \mathrm{Tr}[\rho(\log\rho - \log\sigma^\star)]\; d\Sigma. \tag{5}$$

Theorem 1 (Monotonicity and exponential convergence). *Under axioms (GNS) and (CPTP),* $\delta\Phi/\delta\Sigma(x) \leq 0$. *If in addition (Prim) holds, then* $\rho[\Sigma] \to \sigma^\star$ *exponentially with a rate bounded below by the GNS spectral gap* $\lambda_{\mathrm{gap}} > 0$.

Proof. Modularity (2) implies GNS symmetry [1]: $\langle X, \mathcal{L}(Y)\rangle_{\sigma^\star} = \langle \mathcal{L}(X), Y\rangle_{\sigma^\star}$. Hence $-\mathcal{L}$ is positive in this metric and coincides with the gradient of $D_{\mathrm{rel}}(\cdot\|\sigma^\star)$, yielding monotonicity. Primitivity makes the zero eigenspace one-dimensional and opens a spectral gap; the GNS Poincaré inequality then gives exponential decay of Φ.

Finite-step complete positivity

Theorem 2 (Finite-step CPTP). *If* $\chi_\ell \geq 0$ *and* $\{\gamma_\alpha\}$ *come from positive-definite correlators (Bochner), the finite-step propagator* $e^{\Delta\Sigma\,\mathcal{L}}$ *is CPTP for any* $\Delta\Sigma > 0$.

Proof. The GKLS form ensures complete positivity of infinitesimal maps with Kossakowski matrix positive semidefinite [2,3]. The kernel χ_ℓ makes the finite-step map a convex average of CP maps. Trace preservation follows from the Lindblad form.

Primitivity and uniqueness of the attractor

Theorem 3 (Unique fixed point). *If* $\{F_\alpha\}$ *generates the full local algebra, the semigroup is primitive:* ker $\mathcal{L} = \mathbb{C}\sigma^\star$ *and the stationary state is unique.*

Proof. Irreducibility implies that the commutant of $\{F_\alpha, F_\alpha^\dagger\}$ is $\mathbb{C}\mathbb{I}$; standard quantum semigroup theory then yields uniqueness of the fixed point and a positive spectral gap.

TS integrability and causality

Theorem 4 (Foliation independence). *If* $[\mathcal{L}(x), \mathcal{L}(y)] = [H_\perp(x), \mathcal{L}(y)] = 0$ *for* $(x-y)^2 < 0$ *and* $H_\perp$ *satisfies the hypersurface deformation algebra, then the TS evolution is independent of the chosen foliation* Σ.

Proof. Adapt Schwinger's argument: local commutators integrate to zero over spacelike-separated elements, ensuring path-independence of the ordered exponential along deformations of Σ.

Conservation and Einstein–Langevin coupling

Theorem 5 (Local conservation). *With conserved noise $\xi_{\mu\nu}$ and fluctuation–dissipation relations, and including the Lamb-shift H_{LS}, the renormalized stress tensor satisfies $\nabla_\mu \langle T^\mu{}_\nu \rangle_\rho = 0$ and (4) is consistent with Bianchi identities.*

Falsifiability and quantitative predictions

We outline protocols that access the modular structure and the conserved stochastic back-reaction:

1. **Entropy production bound in a qubit.** Engineering {*Fa*} to be the full set of matrix units on a superconducting qubit makes the dynamics primitive. The relative-entropy half-life obeys $t_{1/2} \leq \frac{\ln 2}{2\lambda_{\text{gap}}}$. For $\lambda_{\text{gap}} = 0.12\,\text{s}^{-1}$ we predict $t_{1/2} \approx 2.89\,\text{s}$.

2. **Equilibrium test of the unified target.** For a 5 GHz qubit at T = 50 mK, the KMS factor is $\beta\hbar\omega \approx 4.80$ and the excited-state fraction is $p_e \approx 0.0081$. Any deviation explained by $\eta > 0$ in (3) can be estimated by tomography.

3. **Choi test of finite-step CPTP.** Reconstruct the Choi matrix of the propagator over a finite TS "slab"; positivity must hold within uncertainties fixed by the noise kernel.

4. **Optomechanical probe of conserved noise.** For a membrane of mass10^{-11}kg at f_m = 100 kHz, Q = 10^6, the on-resonance displacement noise floor is $\sqrt{S_{xx}} \approx 1.3 \times 10^{-18}\ \text{m}/\sqrt{\text{Hz}}$ at room temperature; a conserved Einstein–Langevin contribution at the level of 10^{-4} of thermal noise would be marginally resolvable with state-of-the-art interferometry.

Limiting regimes

The principal asymptotic limits of CUP-Ω* and the corresponding recovered theories are summa- rized in Table 1.

Figures (TikZ/PGFPlots)

METHODS

GNS detailed balance and modular jumps. Choosing $L_\alpha = \sigma_\star^{1/2} F_\alpha \sigma_\star^{-1/2}$ makes $\mathcal{L}$ self-adjoint in the GNS inner product and pins $\sigma^\star$ as the unique fixed point under (Prim). This identifies the flow with a gradient flow for $D_{\text{rel}}(\cdot \| \sigma^\star)$ in the sense of quantum information geometry [1] (Figure 1).

Finite-step CPTP with Bochner kernels. Bochner positivity guarantees the Kossakowski matrix is positive semidefinite; the finite-step propagator is a convex mixture of infinitesimal CPTP maps, hence CPTP (Figure 2).

Einstein–Langevin consistency. Conserved noise ensures $\nabla_\mu \langle T^\mu{}_\nu \rangle_\rho = 0$; Bianchi identities then ensure compatibility of (4) (Figure 3).

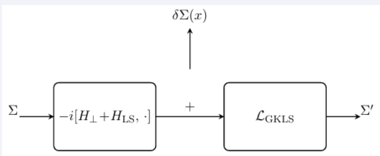

Figure 1 Local TS block: unitary plus modular GKLS applied on a surface element.

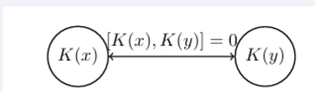

Figure 2 Local integrability under spacelike separation

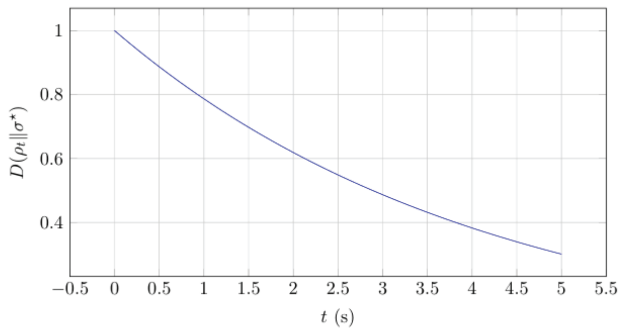

Figure 3 Typical Lyapunov descent of relative entropy with rate bound $2\lambda_{gap}$.

Data availability: No datasets were generated or analysed for this theoretical study.

Code Availability: All LaTeX/TikZ/PGFPlots code to reproduce the manuscript is included in the accompanying project.

Author Contributions: VMG conceived the CUP-Ω∗ framework, developed the mathematical proofs and wrote the manuscript.

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